Plato’s *Timaeus* and the Intervals Used in Traditional Music of the Middle East

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Abstract: This brief study aims to examine the relation between the theory of Pythagoras for the musical ratios (especially as described in Plato’s *Timaeus*) and the intervals used in the traditional Persian music. It can also be generalized and applied to other forms of traditional music in the Middle East. In these forms of traditional music, there are intervals which are slightly different from the standard intervals of the Western music. The study aims to show that these intervals can naturally be deduced from the ideas propounded in Plato’s *Timaeus*.

The Pythagorean Scale

Pythagorean Scale has been derived from his profound discovery according to which most important musical intervals[3] are the ratios between pairs from numbers 1, 2, 3 and 4. In his *Timaeus*, Plato describes the method of Pythagoreans for constructing a scale as follows: initially, he puts between numbers 1 and 2 (octave) ratios $\frac{4}{3}$ and $\frac{3}{2}$ (called perfect fourth and perfect fifth), stating that these ratios give rise to a new interval the ratio of which is $\frac{3}{2} \div \frac{4}{3} = \frac{9}{8}$ (= a tone). Afterwards, he, using this new ratio, fills the spaces between 1 to $\frac{4}{3}$ and $\frac{3}{2}$ to 2, and leaves an

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3- The “interval” between two numbers x and y is the ratio x/y. In a similar way, the interval between two notes is the ratio of their frequencies.

interval in each part which is equal to $\frac{4}{3} \div (\frac{9}{8})^2 = \frac{256}{243}$ (= small Pythagorean semi-tone)[4]:

$$1 \quad \frac{9}{8} \quad \left(\frac{9}{8}\right)^2 \quad \frac{4}{3} \quad \frac{3}{2} \quad \frac{3}{2}\times\frac{9}{8} \quad \frac{3}{2}\times\left(\frac{9}{8}\right)^2 \quad 2$$

Plato himself describes it thus: "... these links gave rise to intervals of $\frac{3}{2}$, $\frac{4}{3}$ and $\frac{9}{8}$ within the original intervals and he went on to fill up all the intervals of $\frac{4}{3}$ with intervals $\frac{9}{8}$, leaving over in each fraction. This remaining interval of the fraction had its terms in the numerical proportion of 256 to 243" (*Timaeus* 35b). The above-mentioned detail can be considered as an explanation for this passage from *Timaeus*. If the frequency of the note Do is 1, the next ratios will respectively be the frequency of the notes Re, Mi, Fa, Sol, La, Si and 2 will be the ratio of the note Do in the next octave:

1	$\frac{9}{8}$	$\left(\frac{9}{8}\right)^2$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{3}{2}\times\frac{9}{8}$	$\frac{3}{2}\times\left(\frac{9}{8}\right)^2$	2
Do	Re	*Mi*	*Fa*	*Sol*	*La*	*Si*	*Do*

There are seven intervals between the successive notes of the scale above: five are equal to $\frac{9}{8}$ and two, equal to $\frac{256}{243}$:

4- It is worth mentioning that there is also another Pythagorean semi-tone which is the interval between two notes with the same names, e.g. Re and Re(#).

$$1 \underbrace{\qquad}_{\frac{9}{8}} \frac{9}{8} \underbrace{\qquad}_{\frac{9}{8}} \left(\frac{9}{8}\right)^2 \underbrace{\qquad}_{\frac{256}{243}} \frac{4}{3} \underbrace{\qquad}_{\frac{9}{8}} \frac{3}{2} \underbrace{\qquad}_{\frac{9}{8}} \frac{3}{2}\times\frac{9}{8} \underbrace{\qquad}_{\frac{9}{8}} \frac{3}{2}\times\left(\frac{9}{8}\right)^2 \underbrace{\qquad}_{\frac{256}{243}} 2$$

There is also **“**Pythagorean comma**”**, which is the interval between Si# and Do; its ratio is $\frac{3^{12}}{2^{19}}$, and a tone is approximately 8.69 commas.

“Quarter tone” of the Persian Music

Now, let us consider the intervals used in the traditional Persian music in comparison with the theory of Pythagoras. In traditional music of Iran, in addition to the tone and semi-tone, there exist other intervals which are slightly different, and are referred to as “quarter tones” (in fact, these intervals are slightly smaller than the semi-tone and this difference is referred to as “quarter tone”).

During the first decades of the 20th century, Ali-Naqi Vaziri, a prominent Iranian musicologist, tried to formulate a theory through generalization of equal-tempered scale and asserted that it is exactly $\frac{1}{4}$ of a “whole tone” (in equal-tempered scale). He considered the signs ⚐ (Koron) and ⧣ (Sori), respectively, for “a quarter tone before” and “a quarter tone after”:

Re(b) Do(#) Re(⚐) Re Re(⧣) Mi(b) Re(#)

But the theory is not verified by experimental measurements. In fact, the measurements performed by Sāsān Sipantā (1998) and Hormoz Farhat (1990) revealed that ⚐ and ⧣ are approximately equal to 3 and 2 commas. Sipantā (1998) and Farhat (1990) themselves performed the measurements using the cent unit. Each cent is equal to $\frac{1}{1200}$ octave; the measurements revealed that  and  are approximately equal to 70 and 45 cents. The table below provides sizes of some important intervals in cents and Pythagorean commas:

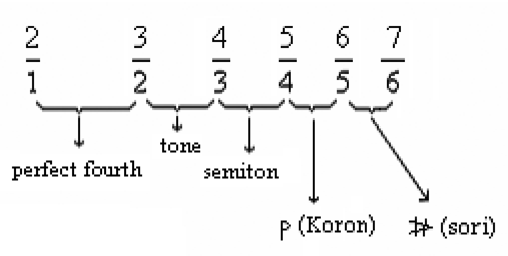
Ratio	Cent	Pythagorean comma
$\frac{3}{2}$	701.9	29.9
$\frac{4}{3}$	498.0	21.2
$\frac{9}{8}$	203.9	8.69
$\frac{256}{243}$	90.2	3.8
$\frac{3^{12}}{2^{19}}$	23.46	1

Generalization of Pythagorean Scale

From the mathematical point of view, natural extension of sequence $\frac{2}{1},\frac{3}{2},\frac{4}{3}$ (Pythagorean ratios) is the sequence $\frac{2}{1},\frac{3}{2},\frac{4}{3},\frac{5}{4},\frac{6}{5},\ldots,\frac{n+1}{n},\ldots$ Plato's method teaches us that in addition to ratios $\frac{2}{1},\frac{3}{2}$ and $\frac{4}{3}$, the intervals between them are also important in the structure of musical scales (for example, $\frac{9}{8}$ is the interval between $\frac{3}{2}$ and $\frac{4}{3}$) and these ratios can be used for filling the larger intervals (for example, filling 1 to $\frac{4}{3}$ by $\frac{9}{8}$ in *Timaeus*). New intervals which appear in the sequence $\frac{2}{1},\frac{3}{2},\frac{4}{3},\frac{5}{4},\frac{6}{5},\ldots,\frac{n+1}{n},\ldots$ are correspondingly:

$$\frac{4}{3}\div\frac{5}{4}=\frac{16}{15}$$
$$\frac{5}{4}\div\frac{6}{5}=\frac{25}{24}$$
and
$$\frac{6}{5}\div\frac{7}{6}=\frac{36}{35}$$

$\frac{16}{15}$ is a kind of semi-tone but $\frac{25}{24}$ and $\frac{36}{35}$ are clearly smaller than a semi-tone. A simple computation shows that $\frac{25}{24}\cong 70.6\ \ cent$ and $\frac{36}{35}\cong 48.7\ \ cent$. These results are consistent with the experimental outcomes for the intervals p and ⧣ which were correspondingly 70 and 45 cents. Therefore, one may conclude that the intervals used in the Persian traditional music may be obtained from the natural extension of the theory propounded by Pythagoras. One may also present this through the beautiful mathematical pattern below:



The aesthetic value and essential simplicity of this mathematical structure as well as the experimental outcomes seem to verify the authenticity of this theory. It also seems that this theory may also be applied to other forms of traditional music throughout the Middle East.

Modulation and Consistency

One may begin a scale by any note. For example, Do major is:

Do Re *Mi* *Fa* *Sol* *La* *Si* *Do*

And Sol major is:

Sol *La* *Si* *Do* Re *Mi* *Fa*# *Sol*

Certainly, the intervals between successive notes must be the same in the two scales and it is true because in Sol major, a small semi-tone (for example, the interval between Mi and Fa) plus a large semi-tone (for example, the interval between Fa and Fa#) is equal to $\frac{9}{8}$ (= tone) which is equal to the interval between La and Si in Do major. One may call this "consistency". There also exists a similar concept in the generalized structure of the Persian traditional music. For example, Humayūn (a scale in the Persian traditional music) is:

Do Re[koron] *Mi* *Fa* *Sol* *La* *Si*b *Do*

If one begins this scale from note Mi, then:

Mi *Fa*[sori] *Sol*# *La* *Si* *Do*# Re *Mi*

The intervals between the first two degrees must be the same in Do-Humayūn and Mi-Humayūn; if one shows the values of Koron and Sori by x and y (using the "comma" unit), then:

$$8.6 - x = 3.8 + y$$
$$\Leftrightarrow x + y = 4.8$$

(Remember that the values of $\frac{9}{8}$ and $\frac{256}{243}$ in comma unit were, correspondingly, equal to 8.6 and 3.8), and this is valid with an acceptable approximation, since:

$$x + y \approx 3.0 + 2.0 = 5$$

The difference (0.2 comma) is too small to be recognized by the aural sense.

Conclusion

As seen above, numbers and proportions between their pairs are of essential importance in musical harmonies. According to Pythagoras and Plato, the soul, before being exiled to earth, had heard the heavenly harmonies, thus the remembrance of these archetypal melodies may be awakened in the soul through listening to music. But, how can music awaken these harmonies if it did not exist there (in the soul) from the beginning? In fact, as Plato explains it in his *Timaeus*, every human being is essentially made by these harmonic proportions. Music is a subdivision of art and art is a certain manifestation of Beauty. As Frithjof Schuon (1990, p. 177), a prominent Swiss-German philosopher, states: “Beauty, being perfection, is regularity and mystery; it is through these two qualities that it stimulates and at the same time appeases the intelligence and also a sensibility which is in conformity with the intelligence.”

The above brief study aims at describing an aspect of “regularity” in the play of “mystery-regularity” interacting at the very foundation of every musical harmony. The aspect of “mystery” is also manifested through so many other components such as “melody” *et cetera*. As the same author writes: “The elements of beauty, be they visual or auditive, static or dynamic, are not only pleasant, they are above all true and their pleasantness comes from their truth: this is the most obvious, and yet the least understood truth of aesthetics. Furthermore, as

Plotinus remarked, every element of beauty or harmony is a mirror or receptacle which attracts the spiritual presence to its form or colour, if one may so express it" (Schuon, 1990, p. 180).

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